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Life Sciences Special Issue

*In this **special issue**, explore the application of mathematics and computational science to numerous topics in the life sciences.*

Figure 2. *Scout bees apply a head-butt stop signal to dancers promoting alternative nest sites. Image courtesy of James Nieh.*

In an article titled "Multi-Agent Decision-Making in Design" on page 6, Paul Davis reports on Naomi Leonard's use of nonlinear dynamics to design multi-agent control systems, taking cues from collective animal behavior.

Understanding Knowledge Networks in the Brain

By Matthew R. Francis

One strength of the human mind is its ability to find patterns and draw connections between disparate concepts, a trait that often enables science, poetry, visual art, and a myriad of other human endeavors. In a more concrete sense, the brain assembles acquired knowledge and links pieces of information into a network. Knowledge networks also seem to have a physical aspect in the form of interconnected neuron pathways in the brain.

During her invited address at the 2018 SIAM Annual Meeting, held in Portland Ore., last July, Danielle Bassett of the University of Pennsylvania illustrated how brains construct knowledge networks. Citing early 20th century progressive educational reformer John Dewey, she explained that the goal of a talk—and learning in general—is to map concepts from the speaker/teacher's mind to those of his or her listeners. When the presenter is successful, the audience gains new conceptual networks.

More generally, Bassett explored how humans acquire knowledge networks, whether that process can be modeled mathematically, and how such models may be tested experimentally. Fundamental research on brain networks can potentially facilitate the understanding and treatment of conditions as diverse as schizophrenia and Parkinson's disease.

In mathematical terms, a network is a type of graph: a set of points connected by lines. The particular model for knowledge networks is based on the assumption that humans essentially experience phenomena as discrete events or concepts arranged sequentially in time. Each of these is modeled as a node in a graph, with lines called "edges" linking them together. The edges represent possible transitions between the events or concepts; a particular graph thus describes how knowledge networks interconnect ideas. The first big question for this model is whether optimal pathways of connectivity that maximize learning exist.

Hot Thoughts and Modular Thinking

To address this problem, Bassett and her collaborators constructed a knowledge network consisting of nodes that represent random stimuli. Each node was connected to the same number of other nodes—cre-

See **Knowledge Networks** *on page 4*

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Mathematical Models of Cell Invasion Track Spatial Position and Progression through the Cell Cycle

By Matthew J. Simpson

Spatial spreading of cell populations is crucial for embryonic development, tissue repair, and malignant invasion. During embryonic development, various types of tissues and organs with different functions evolve from an initially homogeneous population of cells in the very early embryo. Normal development requires that these structures arise with the correct cell types in the appropriate three-dimensional geometry, a process that necessitates simultaneous migration and growth of cell populations. Abnormal cell migration during adulthood is associated with pathological conditions like cancer invasion and metastasis. While motility of individual cells is crucial for population-level spreading, cell proliferation plays an important and sometimes overlooked role in driving spatial expansion of cell populations [4].

Previous researchers have employed both continuum differential-equation models and discrete models that use lattice-based random walks, lattice-free random walks, and cellular automata to mathematically model spatial spreading in cell populations. Continuum mathematical models of combined cell motility and proliferation are often based on the Fisher-Kolmogorov equation [3, 4, 6]. We can write this equation in one dimension as

$$\frac{\partial s}{\partial t} = D\frac{\partial^2 s}{\partial x^2} + \lambda s\left(1 - \frac{s}{K}\right), \qquad (1)$$

where $s(x,t) > 0$ represents cell density, $D > 0$ is cell diffusivity, $\lambda > 0$ signifies the cell proliferation rate, and $K > 0$ denotes the environment's carrying capacity density. This simple mathematical model supposes that cells undergo an unbiased random walk and proliferate logistically so that some initial density will evolve—through a combination of diffusion and carrying-capacity limited growth—to produce moving fronts. Such fronts lead to the colonization of initially-vacant regions that will eventually reach carrying capacity density K. For certain initial and boundary conditions, (1) can yield travelling wave solutions [4]. This type of modelling framework has successfully described a range of phenomena related to collective cell spreading and cell invasion, from the study of simple *in vitro* scratch assays to *in vivo* malignant brain tumour progression.

In 2008, the introduction of a novel experimental methodology called the Fluorescent Ubiquitination-based Cell Cycle Indicator (FUCCI) [5] enabled real-time analysis of the cell cycle. FUCCI provides spatial and temporal information about the cell cycle's progression through fluorescent probes that glow red when the cell is in the first gap (G1) phase of the cycle and green when in the synthesis (S), second gap (G2), or mitotic/cell division (M) phases [5, 9].

Figure 1a depicts still images from a time-lapse movie of a FUCCI-transduced melanoma cell that illustrate these colour changes during the cell cycle. A visual comparison of the phase contrast image of a C8161 melanoma spheroid (see Figure 1b) and a corresponding FUCCI-transduced spheroid (see Figure 1c) confirms that the FUCCI system reveals a great deal about the relationship between spatial location and the cell cycle. For example, freely cycling cells—indicated by the mixture of red and green fluorescence—are visible on the FUCCI spheroid's outer shell. Upon looking deeper into the spheroid, we observe cells that are largely arrested in the G1 (red) phase, presumably due to a lack of nutrients such as

See **Cell Cycle** *on page 2*

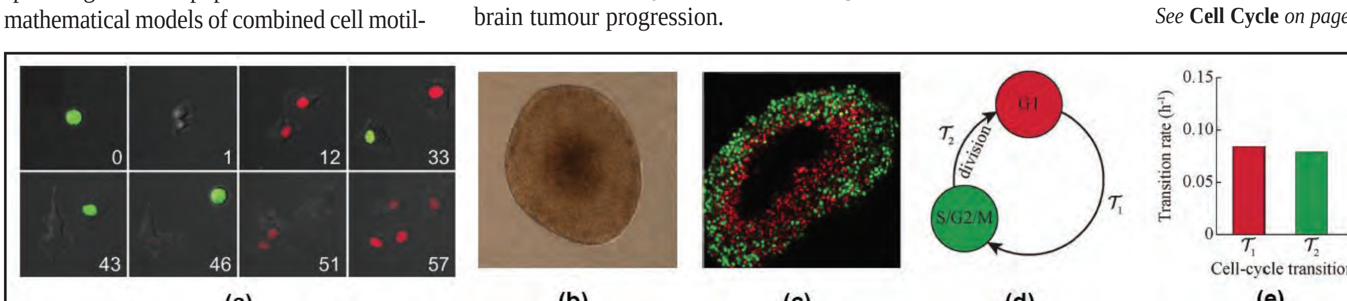

Figure 1. *Experimental images and conceptual model for the cell cycle as revealed by FUCCI labelling.* **1a.** *Time-lapse images depicting the progression of three generations of FUCCI-transduced C8161 melanoma cells through the cell cycle. The numbers in each subfigure indicate the time in hours.* **1b.** *Phase contrast image of a C8161 melanoma spheroid with an approximate diameter of 0.5 millimeters.* **1c.** *Image of a slice of a C8161 melanoma spheroid of similar size to 1b where the cells are transduced with FUCCI.* **1d.** *Conceptual model of FUCCI technology.* **1e.** *Experimental data presenting quantification of the rate of red-to-green and green-to-red transitions for the C8161 melanoma cell line. 1a courtesy of [2], 1b and 1c courtesy of [1], 1d and 1e courtesy of [8].*

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Cell Cycle

Continued from page 1

oxygen. The spheroid's centre completely lacks fluorescence; this indicates the presence of a necrotic core. Unsurprisingly, FUCCI technology has revolutionized the analysis of cell proliferation and has applications in experimental models of cancer, stem cell, and developmental biology [9].

We have recently sought to develop and validate continuum models of cell invasion for use with FUCCI technology [7, 8]. We employ FUCCI-transduced C8161 melanoma cells and perform a series of scratch assays to observe cell migration; these assays involve creating a "scratch" in a cell monolayer and capturing images close to the scratch at regular intervals. Next we attempt to quantitatively describe these experiments with a new continuum model — a generalisation of the Fisher-Kolmogorov equation.

Figure 2a shows a population of FUCCI-transduced C8161 melanoma cells on a two-dimensional substrate following the creation of a 0.5 millimeter artificial scratch in the population. Figure 2d displays the same experiment after 18 hours; now we see that the cells have moved into the initially-vacant region while continuing to progress through the cell cycle. To model these experiments, we suppose that the cell population—with total density $s(x,t)$—consists of two subpopulations, yielding $s(x,t)=v_r(x,t)+v_g(x,t)$. Here $v_r(x,t)$ is the red cell density in the G1 phase and $v_g(x,t)$ is the green cell density in the S/G2/M phases. Assuming that both subpopulations undergo diffusive migration and proceed through the cell cycle, we encode this information into a generalisation of the Fisher-Kolmogorov equation given by

$$\frac{\partial v_r}{\partial t}=\mathcal{D}_r\frac{\partial^2 v_r}{\partial x^2} -\kappa_r v_r+2\kappa_g v_g\left(1-\frac{s}{K}\right), \qquad (2)$$

$$\frac{\partial v_g}{\partial t}=\mathcal{D}_g\frac{\partial^2 v_g}{\partial x^2} -\kappa_g v_g\left(1-\frac{s}{K}\right)+\kappa_r v_r, \qquad (3)$$

where $\mathcal{D}_r>0$ is the diffusivity of cells in the G1 phase, $\mathcal{D}_g>0$ is the diffusivity of cells in the S/G2/M phases, $\kappa_r>0$ is the rate of cell transition from the G1 phase to the S/G2/M phases, and $\kappa_g>0$ is the rate of cell transition from the S/G2/M phases to the G1 phase.

Terms proportional to κ_r model the red-to-green transition. This transition is conservative because the positive term in (3) balances the negative term in (2). Terms proportional to κ_g model the green-to-red transition, which is not conservative due to the factor of two in (2) that reflects this transition's association with cell division. This also explains why we include the nonlinear logistic parameter in terms proportional to κ_g, since this event would require sufficient space to accommodate the daughter agent. Our result is directly analogous to the per capita proliferation rate in the Fisher-Kolmogorov equation existing as a linearly decreasing function of total density.

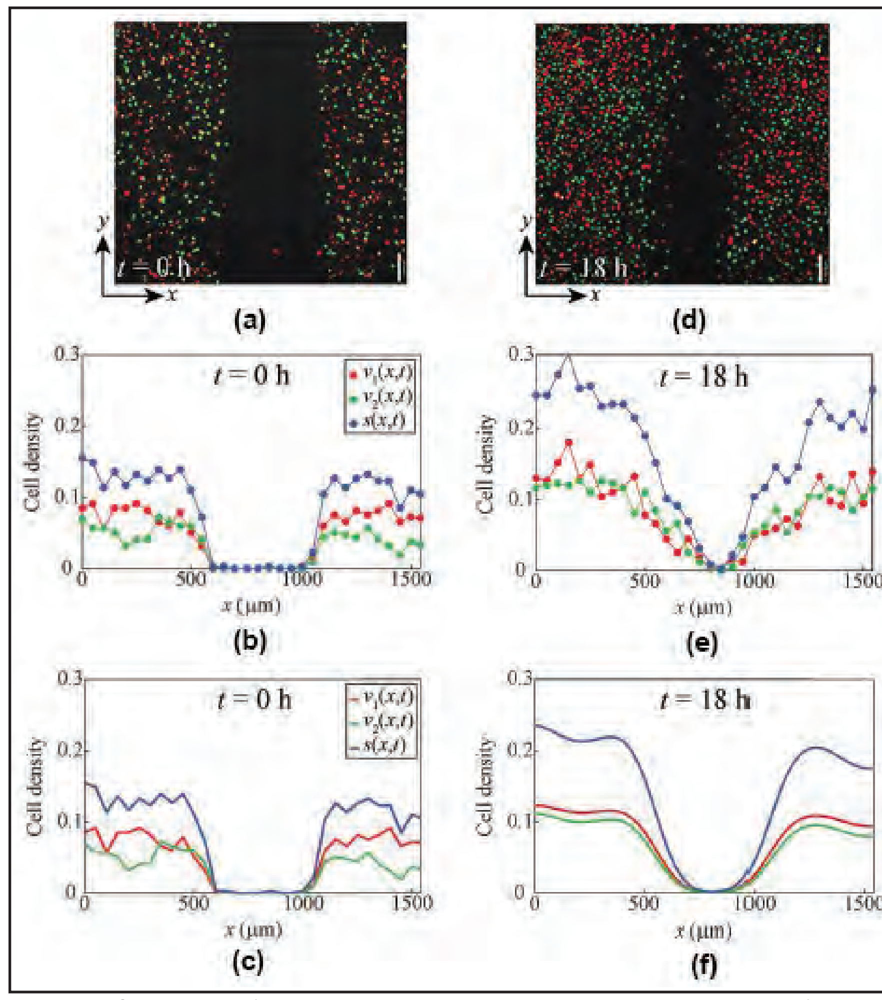

Figure 2. *Comparison of experiential images, experiential data, and solutions of the new mathematical model for scratch assays with FUCCI-transduced melanoma cells.* **2a-2c.** *Initial condition for experimental modelling comparison* $t=0h$. **2d-2f.** *Comparison of experimental observation and modelling prediction* $t=18h$. *2a and 2d display images of a scratch assay with C8161 melanoma cells where the scratch's initial width is approximately 0.5 millimeters. We extract density profiles at* $t=0h$ *from 2a and plot them in 2b and 2c, where the red profile corresponds to cell density in the G1 phase, the green profile corresponds to cells in the S/G2/M phases, and the black profile corresponds to total density. We also extract density profiles at* $t=18h$ *from 2d and plot them in 2e. Solutions of (2) and (3) in 2f—obtained with* $\mathcal{D}_r=400\mu\text{m}^2/h$, $\mathcal{D}_g=400\mu\text{m}^2/h$, $\kappa_r=0.084/h$, $\kappa_g=0.079/h$, *and* $K=0.004$ *cells/*μm^2*—compare very well with experimental profiles in 2e. Figure courtesy of [8].*

To determine whether (2) and (3) can predict the spatial and temporal dynamics of the scratch assay in Figure 2a, we divide each image of the assay into a series of equally-spaced columns. Counting the number of red and green cells per column and dividing by the column area estimates the total cell density and the density for both the red and green subpopulations (see Figure 2b). The density profile in 2e portrays the system's state after 18 hours. Two main outcomes are apparent: the initially-vacant region is now occupied, and the cell cycle continues as the density of cells away from the initially-vacant region approximately doubles. Solving (2) and (3) for the initial condition in Figures 2b and 2c generates the prediction in Figure 2f, which matches the experimental observation in Figures 2d and 2e remarkably well.

In summary, the continuum modelling framework in (2) and (3) shows promise in quantifying experimental investigations with FUCCI technology. Researchers can extend this type of modelling platform in multiple ways, such as in the context of different geometries and coordinate systems. One could also develop an analogous set of modelling tools based on a discrete random walk with exclusion and subsequently apply these tools to reveal both population-level and individual-level properties [7]. We anticipate further refinements in these mathematical modelling tools to keep pace with new advances in experimental cell biology.

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New Results in Stochastic Analysis Using Dynamical Systems Theory

By Lora Billings and Eric Forgoston

With its long history in a variety of fields, stochastic analysis continues to challenge modelers. Noise plays an important functional role in changing a system's dynamics in applications across mathematical finance, physics, and biology [7]. Researchers in multiple disciplines have developed numerous approaches and notations to understand the dynamics beyond numerical simulation. Adding to the complexity, one can model noise as an external source or a collection of random internal processes, and few have considered incorporating either concept into models with time-dependent forcing. Inspired by the work in [2, 4], we find that classical tools from dynamical systems and perturbation theory provide an intuitive way to analyze stochastic epidemiological models with seasonal forcing and predict noise-induced rare events.

Seasonal forcing is important in epidemic modeling because contact rates and other parameters may vary in time in diseases like measles, mumps, and chickenpox. This is evident from the periodic and aperiodic recurrence of such conditions, as observed in pre-vaccine United States data and English and Welsh data [1]. Our recent work considers a general class of dynamical models with internal noise that symbolizes demographic stochasticity, or random interactions of individuals [3]. Our model can represent any group of well-mixed individuals from the nano- to macro-scale. We study disease dynamics in a human population that requires seasonality to accurately capture the force of infection, and focus on internal random dynamics that can induce a rare, spontaneous disease eradication.

To understand random dynamics, one can use a master equation that describes the probability of observing a given number of individuals in a population at a certain time. As the master equation generally cannot be solved analytically, we use a Wentzel-Kramers-Brillouin-Jeffreys (WKBJ), or eikonal, approximation to the master equation. This ansatz approximates the probability density function for the system dynamics and leads to the development of a Hamiltonian system. The Hamiltonian's dimensions are twice the dimensions of the original system's mean-field equation due to the conjugate momenta variables, which capture the effects of noise. The benefit is that the transformed system is deterministic and lends itself to dynamical systems theory. For instance, standard methods can identify steady states, which correspond to the metastable states in the original stochastic system.

As an example, consider a time-dependent susceptible-infected-susceptible (SIS) model consisting of a finite population of N individuals divided into two groups: susceptibles and infectives. The flux terms for these groups are as follows. An individual is born susceptible, assuming a constant birth rate. Through contact modeled by a mass action approximation, he/she may become ill and be classified as infectious. We suppose that the contact rate varies throughout the year in a periodic fashion, approximated by the time-dependent function $\beta(t) = \beta_0(1+\delta\cos(2\pi t))$, where t is time, β_0 is the average contact rate, and δ is a small parameter. Assuming an average recovery rate, the individual recovers and returns to the susceptible group. While removal by natural death is possible from both groups, we presume the absence of disease-related deaths in this model.

Figure 1 offers a numerical realization of this stochastic time-dependent SIS model for parameters that support the disease's persistence. We use a Monte Carlo algorithm to evolve the population in time [5]. In Figure 1, the population persists near what appears to be an oscillating steady state and eventually drops to zero via spontaneous extinction. Our goal is to develop a method to analytically predict the average time of occurrence for an extinction event. Numerous simulations reveal the existence of many possible escape paths, with extinction taking place at different times. Aggregation of simulation data demonstrates a path along which extinction is most likely to occur. This optimal path coincides with the manifold connecting steady states in the corresponding Hamiltonian system. Identifying the action along the optimal path lets researchers determine the mean time of escape from a steady state or the average time to an extinction event.

The analytical approach uses the WKBJ approximation $\rho = e^{-N\mathcal{S}(i,t)}$ for the probability distribution in the master equation, for which $\mathcal{S}$ is the action and the state variable i is the scaled infective population. The leading order of a Taylor series expansion of the scaled master equation results in a Hamilton-Jacobi equation with an effective Hamiltonian, for which we analyze the zero-energy solutions. The WKBJ approximation introduces a conjugate momentum variable $p = \partial\mathcal{S}(i,t)/\partial i$, which doubles the system's dimension but reveals additional dynamics that stem from stochasticity.

The advantage of recasting the problem in terms of the conjugate momentum is that it reveals a higher-dimensional Hamiltonian system with steady states classified as saddles and centers. This allows a trajectory to escape from the deterministically stable endemic limit cycle through the expanded p space (see Figure 2, on page 5). Specifically, there exists a heteroclinic trajectory connecting it to the fluctuational extinction state for which $i=0$ and $p \neq 0$.

If one rewrites the Hamiltonian using a linear expansion in δ, the Poincaré-Birkhoff fixed point theorem guarantees

See **Stochastic Analysis** *on page 5*

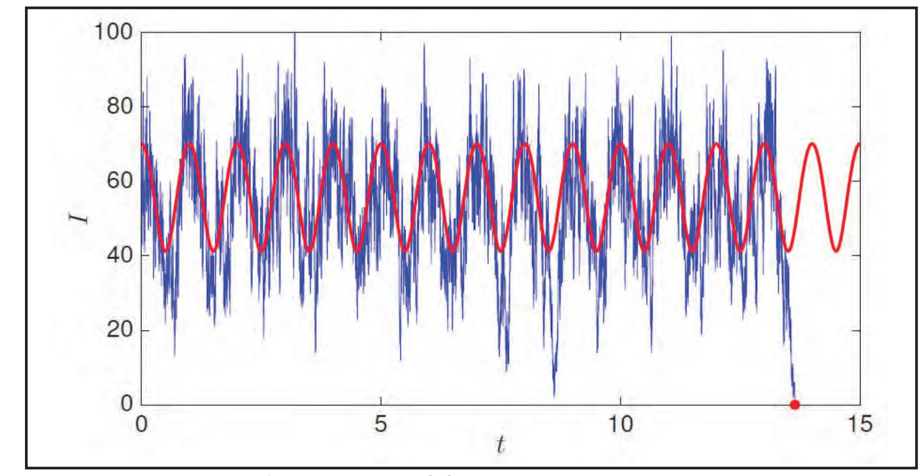

Figure 1. *A realization of the stochastic SIS model with seasonality exhibiting an extinction event. We plot the number of infectives (I) in time (t) in blue. Note how the graph follows the red curve, which represents the endemic limit cycle given by the solution to the mean field equation ($p=0$, no noise). Image courtesy of [3].*

Updates from the December SIAM Board Meeting

By James Crowley

At its December meeting, the SIAM Board of Trustees took many significant actions of interest to members, including changes in membership options and approval of a new SIAM activity group (SIAG), conference, and section.

Going Green

Of note is the move towards a greener option with cost savings. SIAM membership has always included a print copy of *SIAM Review*. In its desire to reduce SIAM's carbon footprint and save money, the Board agreed to make receipt of the print copy of *SIAM Review* an opt-in selection. Members who do not actively indicate their preference to continue to receive the print copy in their next renewal will not receive it. Of course, all members will retain access to the online version as part of their membership.

Along the same lines, the Board moved to eliminate paper copies of full programs from conferences whenever practical (except for the Program-at-a-Glance, which will still be available in print). The online program and the mobile app (for those conferences that provide apps) will allow attendees to find information on sessions and talks at future SIAM meetings.

Activity Groups and Sections

The Board also approved creation of the SIAM Activity Group on Applied and Computational Discrete Algorithms (SIAG/ACDA). This SIAG grew in part out of a series of SIAM workshops on Combinatorial Scientific Computing but has expanded coverage to include more subjects. SIAG/ACDA fosters research on the design, theoretical analysis, computational evaluation, and deployment of algorithms for combinatorial problems formulated from applications. It complements—but does not replace—the SIAG on Discrete Mathematics, which will continue operation according to its approved charter. A biennial SIAM conference will be organized by the SIAG. Further details about SIAG/ACDA will be forthcoming.

Besides this new SIAG, the Board recently authorized the creation of the SIAM Conference on Mathematics of Data Science. The inaugural meeting will occur in the spring of 2020 and be co-located with—but independent of—the SIAM Conference on Data Mining. The dates and location are still being finalized at the time of this writing. Like a standard SIAM conference, the data science conference will feature minisymposia and invited talks. Its topical coverage will be similar to areas served by the new *SIAM Journal on the Mathematics of Data Science*,[1] which began accepting papers in 2018.

In the fall of 2018, the Board also approved the SIAM Northern States Section[2] — a new SIAM section that covers a region of the U.S. including the Dakotas, Montana, Utah, and Wyoming. As is standard practice, members of SIAM living in the designated region are considered members of the section. Sections hold regional meetings, workshops, and conferences. The first meeting of the Northern States Section will take place in the fall of 2019. Additional conversation about regional sections—their purpose, funding, and governance—will occur at a later date.

[1] https://www.siam.org/Publications/Journals/SIAM-Journal-on-Mathematics-of-Data-Science-SIMODS

[2] https://www.siam.org/Membership/Sections/Detail/siam-northern-states-section

Tondeur Donation

SIAM leadership wishes to thank Philippe Tondeur (University of Illinois at Urbana-Champaign) for his very generous donation to SIAM. Philippe contributed $100,000 to SIAM—and a similar amount to its sister societies, the American Mathematical Society and the Mathematical Association of America (MAA)—to support the mission of the BIG Math Network,[3] a joint collaboration among several organizations to raise awareness and promote opportunities for mathematics students' employment in business, industry, and government. The BIG Math Network—formerly chaired by Rachel Levy (MAA), then SIAM Vice President for Education—is currently led by Fadil Santosa (University of Minnesota). SIAM is developing programs, in coordination with the BIG Math Network, to carry out Philippe's wishes.

Changes to the Fellows Program

The SIAM Fellows Program has been very popular in recent years in terms of nomination submissions. The large number of nominations necessitated several modifications, including an earlier nomination submission deadline to provide the Fellows Selection Committee with more review time. This year's nominations will close on Wednesday, October 16. To simplify the selection process and ensure fairness, some standardization of nomination packages will also be enforced. See the Fellows website[4] for details pertaining to specifications on page length, font size, etc. SIAM thanks those involved for their hard work in making this a successful program.

[3] https://bigmathnetwork.org/

[4] https://www.siam.org/Prizes-Recognition/Fellows-Program

Looking Ahead

Through a set of ad hoc committees, the Board has started further discussion on a variety of issues, motivated in part by SIAM's deficits over the past few years. *SIAM News* readers will hear more about the results from these committees in future updates.

January 1 was a date of many transitions, as newly-elected members of the Board and Council assumed office and new committee memberships took effect. We would like to thank all outgoing committee members for their service. Two particular transitions are of note: Lisa Fauci (Tulane University), elected in the fall of 2017, takes over as SIAM President after one year as President-Elect. Nick Higham (University of Manchester) becomes Past President for 2019.

We also welcome Richard Moore to SIAM as the new Director of Programs and Services. Richard is replacing Linda Thiel, who retired in late 2018.

Suggestions?

SIAM is happy to receive suggestions from its members. You can do this through the footer of any page of the SIAM website,[5] where an item labeled "Suggestions" links to a form that states, "Whether you want to recommend a member to a leadership position, add your company to our list of career resources, weigh in on our new website, ask a question, or anything else, we want to hear from you." Let us know if you have a question or suggestion. Suggestions made on the site go to me, the executive director, and I ensure that the appropriate person or committee receives and reacts to each message.

James Crowley is the executive director of SIAM.

[5] https://www.siam.org

Knowledge Networks

Continued from page 1

ating a k-regular graph, in graph theory terms—to ensure equal transition probabilities between nodes.

The researchers tested human reactions, assigning key presses or abstract "avatars" to every node. Test subjects "learned" the graph by performing set tasks, and Bassett's team quantified these performances based on the amount of time the subjects spent responding to each task. The experiments involved two types of graphs: graphs in which nodes clustered together in groups and graphs with evenly-connected nodes in a more lattice-like structure. For example, consider a four-regular graph of 15 nodes where each node is connected to four others. The modular graph could simply cluster the nodes into three linked groups of five (see Figure 1), while the lattice-like graph lacks modules [3].

Although both graphs were k-regular, subjects learned the modular graphs more efficiently and performed faster on transitions between nodes within a module than transitions between modules themselves, regardless of the nature of the task. Edges between clusters are called "cross-cluster surprisal" because they had slower reaction times than those within modules.

These results suggest that human minds "lump" concepts together, a premise that is borne out by other psychological tests. Despite having equal transition probabilities for all graph edges, the brain evidently distinguishes between topological "distances" by the type of transition performed in the graph. This indicates that humans implicitly recognize the graph's topology.

To quantify how well test subjects recalled the learned material, Bassett and her colleagues assigned a probability to time interval Δt between when the event actually happened and when the subjects thought it occurred. Drawing on thermal physics, the team associated Δt with the concept of cognitive "free energy," which the brain minimizes to reduce computational resources and recall errors [4].

In this language, the probability P of recall for a given time interval Δt after an event is

$$P(\Delta t) = \frac{1}{Z} e^{-\beta \Delta t},$$

where Z is the partition function

$$Z = \sum_{\Delta t} e^{-\beta \Delta t}$$

and β is the parameter that sets the distribution scale. Bassett suggested that humans operate at a particular "temperature" $T \sim 1/\beta$ that changes over the course of their lives (this is similar to how temperature is defined in information theory). High "temperatures" (limit as $\beta \to 0$) result in a flat probability distribution, which means poor graph recall. Low "temperatures" ($\beta \to \infty$) ensure that the probability drops precipitously for nonzero Δt values, thus corresponding to an accurate memory. For moderate "temperatures," the subjects reproduce the basic graph — but with some errors (see Figure 2).

Continuing with the physics metaphor, this model is akin to the difference between materials at various temperatures. Solids are often highly ordered at low temperatures, with atoms arranged in predictable patterns; raising temperatures destroys the order, giving rise to random and time-variable fluid arrangements.

Networks and Learning

Bassett then presented a larger question: Is it possible to design optimal knowledge networks that help people learn? As is moderately obvious, there is much individual variation among humanity in terms of cognitive function. At the same time, the mind's ability to reconstruct modular patterns seems to indicate a general, shared means of operation. Bassett posed the question as follows: Are the actual, physical neural networks in a brain modular?

To explore this hypothesis, she and her collaborators tested subjects while they were inside a magnetic resonance imaging (MRI) machine. They found that real brain networks are multilayered and dynamic, as opposed (for example) to one static group of cells that routinely corresponds to learning.

More to the point, the flexibility of physical brain networks apparently linked to individual cognitive capabilities. Low flexibility limited subjects' ability to learn and retain information, but Bassett's team noted that certain test subjects with schizophrenia exhibited very high flexibility along with other deficits in function, leading the group to hypothesize an optimal range for flexibility to support cognitive performance [1].

Bassett and her colleagues were interested in connecting their mathematical model with the MRI results to examine a possible correlation between cognitive control and brain dynamics. MRI studies indicate that network control increases as children develop, approaching an asymptotic maximum roughly at age 20 in healthy brains [2].

When discussing cognitive abilities and mental health, researchers must always be cautious about ethical complications. Bassett and her collaborators are prudent to argue that one should handle issues involving brain control with caution in order to prevent misuse [5]. Understanding the way the mind works leads to questions of how and when mind control modification is possible or advisable. While control structures can be beneficial in treating some cognitive conditions resulting from lack of internal control, they can give rise to possible misuses as well (and not just science-fiction-style whole-brain hijacking scenarios).

Bassett presented a therapeutic argument for this avenue of research: if human brains control their network functions in particular ways, introducing external controls to change or enhance certain behaviors might be possible. For conditions like epilepsy or Parkinson's disease, these modifications could be extremely helpful.

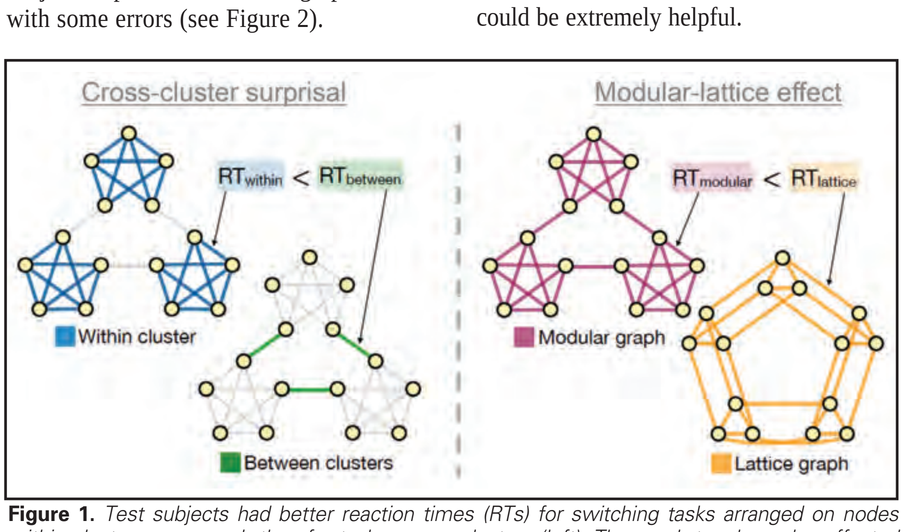

Figure 1. *Test subjects had better reaction times (RTs) for switching tasks arranged on nodes within clusters on a graph than for tasks across clusters (left). The graph topology also affected performance, with better behavior on clustered nodes than nodes with a lattice-like topology (right), even when both graphs had the same number of edges per node. Figure adapted from [4].*

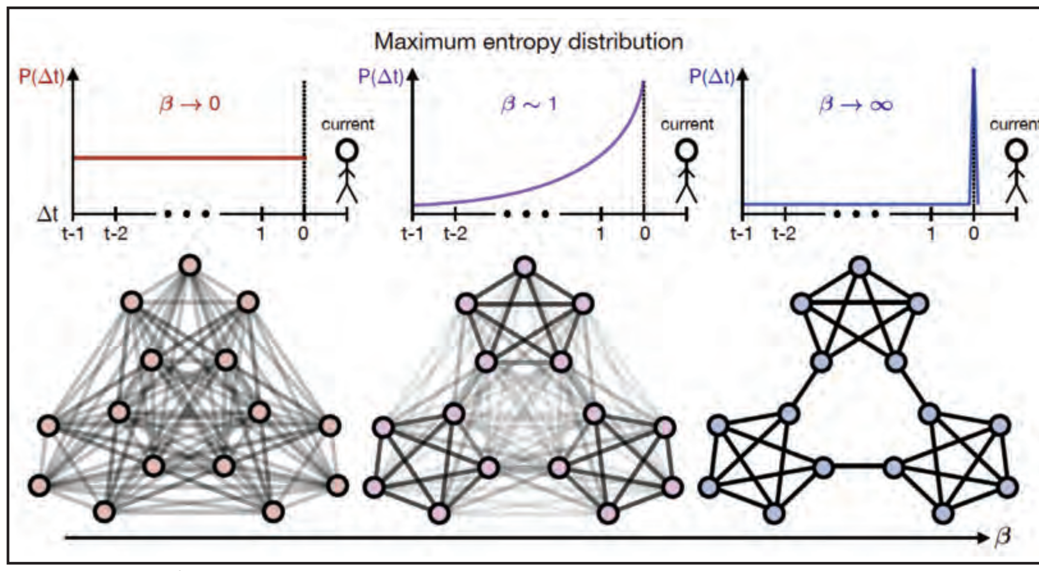

Figure 2. *Recall of tasks can be modeled using statistical mechanics, where the inverse "temperature" β parameterizes the probability of recall over time. High temperature or low β produces a messy graph recall, while lower temperatures represent increased accuracy. The darkness of the graphs' edges indicates how well subjects remember the transitions between tasks on the graph topology from Figure 1. Figure courtesy of [4].*

Ultimately, network models facilitate one's understanding of how the mind, and possibly the brain itself, functions. Thus, researchers could extend many areas of applied mathematics—currently used in information theory, network control, and thermodynamics—to the study of the mind. Such connections satisfy the human impulse to find and transform patterns into something new.

Bassett's presentation is available from SIAM either as slides with synchronized audio or a PDF of slides only.[1]

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Further Reading

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Politics, Policy, Science, and Risk

The Fifth Risk. *By Michael Lewis. W.W. Norton & Company, New York, NY, October 2018. 256 pages, $26.95.*

BOOK REVIEW
By Paul Davis

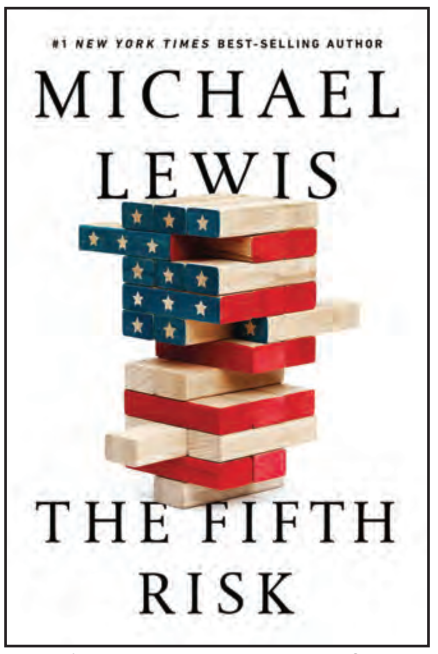
Michael Lewis’s 2010 best-seller, *The Big Short*, describes the global economy’s catastrophic fall onto the sharpened stakes in the pit of subprime mortgages. He tells the tale through the eyes of prescient—if sometimes idiosyncratic—players who were “in the room where it happened,” as Aaron Burr raps to Alexander Hamilton in Lin-Manuel Miranda’s eponymous musical. Lewis’s latest book, *The Fifth Risk*, tells its story of politics, policy, and science with similar energy and an analogous set of engaged and engaging characters. But unlike the widespread economic devastation described in *The Big Short*, politics has not quite pushed the entirety of federal science into the pit. The dazed victim is merely teetering on the edge, punctured and bleeding but not yet fully lacerated.

The Fifth Risk is a quick and gripping read. A prologue describes the willfully determined ignorance underlying the Trump administration’s neglectful transition. Three subsequent chapters tell tales of the aftermath, one each for the U.S. Departments of Energy, Agriculture, and Commerce. For scientists who have spent time in Washington, D.C., reading Lewis’s account is likely akin to sitting down with a John le Carré thriller set in a familiar place. Caught up in the fast-paced plot, they are transported back to locations they know, brain-numbing corridors in monolithic government offices, ideas, policy debates, and talking points. They see skeletons that rattle like ones they might have hauled through those same buildings, even if the flesh that falls from the bones in Lewis’s narrative differs from their own experiences.

The first chapter’s setting in the Department of Energy (DOE) is most likely to produce a le-Carré-esque response among mathematicians. The DOE’s national labs are central to applied and computational mathematics. Many SIAM members have worked in or visited these labs, while others have served as rotators in the department’s research program offices.

Lewis found DOE alumnus John MacWilliams at home five months after he had left the department, his briefing books untouched by the Trump transition team. In 2013, then-Secretary-of-Energy Ernest Moniz (a physicist, not a politician) invited MacWilliams to assess the DOE’s financial risks using his experience in law and investment banking. MacWilliams’s remit quickly expanded to encompass all risks facing the DOE, including nuclear weapons, nuclear power generation, spent fuel disposal, the electric power grid, and more.

Sitting across from MacWilliams at his kitchen table, Lewis sought to emulate the transition briefing that never happened. “Just give me the top five risks I need to worry about *right away*,” he brusquely demanded of MacWilliams. “Start at the top.” MacWilliams wanted to describe an accident with nuclear weapons, but Lewis—like the phantom transition team—lacked a security clearance. Generalities had to suffice. The same constraint limited MacWilliams’s description of his second proposed risk to two words: North Korea.



The Fifth Risk. By Michael Lewis. Courtesy of W.W. Norton & Company.

Despite the security restrictions, MacWilliams and his former colleagues from the DOE said enough to frighten Lewis. They spoke of precarious scenarios involving not limited to—technical failures in the electric grid or storage of radioactive waste. Of course, more than a few SIAM members have studied aspects of both possibilities in depth.

Compared to these Hollywood-style disaster situations, MacWilliams’s “fifth risk,” which lends the book its title, seems anticlimactic. He identifies this titular risk as “project management.” But where is the risk in project management? Are the scientists managing research programs—whether as rotators or permanent staff at DOE laboratories, the National Science Foundation, Office of Naval Research, Army Research Office, Air Force Office of Scientific Research, Department of Defense, etc.—really engaged in perilous work? Was MacWilliams making a bad joke about paper shufflers’ exposure to paper cuts?

Certainly not. Lewis explains that MacWilliams’s “more general point was that managing risks is an act of the imagination.” Humans can respond to a real-time crisis better than they can conceive of one that is forthcoming. “There is another way to think of John MacWilliams’s fifth risk: the risk a society runs when it falls into the habit of responding to long-term risks with short-term solutions,” Lewis writes.

“‘Program management’ is not just program management…It is what you never learned that might have saved you.” It is the best kind of science — conceiving the inconceivable.

With that sobering warning in mind, the remaining chapters on the Departments of Agriculture and Commerce are distressing yet engrossing. Admiration for the work of Lewis’s heroes competes with concern over what could happen without such stalwarts at the ramparts; most of Lewis’s sources are reporting after leaving Washington.

The final section on the Department of Commerce will draw many SIAM readers through its focus on the National Oceanographic and Atmospheric Administration (NOAA). This agency consumes more than half of the Commerce budget to generate and analyze big data—really big data—to predict the weather. Lewis reminds readers that the Department of Commerce also houses the U.S. Patent and Trademark Office and the National Institute of Standards and Technology, home to other active SIAM members.

DJ Patil appears frequently in this chapter. Patil began his career studying dynamical systems and hacking into the National Weather Service to grab weather data for his doctoral research. He was appointed Chief Data Scientist in President Barack Obama’s Office of Science and Technology Policy. Despite Patil’s high-level view of government data, some things still caught him off guard. “In the end, even DJ Patil was shocked by the possibilities that lurked in the raw piles of information the government had acquired.” Lewis writes. “‘I didn’t grasp the scope at first,’ he said.”

“After Trump took office,” Lewis reports, “DJ Patil watched with wonder as the data disappeared across the government.” Links to climate change data evaporated, along with reports of consumer complaints about businesses. Statistics on access to drinking water in Puerto Rico vanished two weeks after Hurricane Maria. Nearly three-quarters of the data tables in the annual FBI crime report disappeared. Links to weather forecasts on the NOAA site moved from prominence to obscurity. All sorts of data that could fuel discovery, insight, and understanding fell from public reach.

You might wonder what awful things could arise from neglecting the Department of Agriculture, Lewis’s third subject. When Lewis asked one recently retired senior official to identify his primary concern, he replied, “Wildfires.” Yet we have heard little about them in the news recently!

The entirety of *The Fifth Risk*, brief as it is, offers compelling accounts of scientists, engineers, and science-savvy government employees struggling to keep the lights on in their hard-won corners of expertise. Lewis’s intense account of politics’ heavy hand falling on government science should draw more civic-minded scientists into government. The SIAM members and their colleagues who heed Lewis’s call are unlikely to become central characters in his next book. Nonetheless, they will have a memorable opportunity to learn and serve during a critical time in our shared professions.

Paul Davis is professor emeritus of mathematical sciences at Worcester Polytechnic Institute. He served as an assistant program manager in applied mathematics at the Army Research Office from 1974 to 1975, where his duties included managing proposal evaluations and advocating for the Mathematics Research Program. As a Jefferson Science Fellow from 2007 to 2008, Davis was a Senior Science Advisor in the Bureau of East Asian and Pacific Affairs of the U.S. Department of State. His primary responsibility was establishing the structure of an agreement for scientific cooperation between the United States and the 10 nations of the Association of Southeast Asian Nations.

Stochastic Analysis

Continued from page 3

existence of the perturbed hyperbolic points. The presence of a corresponding heteroclinic trajectory depends on the optimal correction to the action, or the minimization of $\mathcal{S}$ with respect to initial time t_0. Finding the zeros of the derivative of $\mathcal{S}$ yields an integral of a Poisson bracket of the unperturbed Hamiltonian and the time-dependent Hamiltonian evaluated on the unperturbed optimal path; this integral is the perturbed problem’s Melnikov function. A sufficient condition for the perturbed optimal path’s existence is the Melnikov function’s possession of simple zeros. We can numerically evaluate this equation to identify a periodic set of zeros, which determines the phase for the optimal correction to the action.

While we cannot explicitly solve the time-dependent Hamilton’s equations, we can numerically approximate the heteroclinic manifold via the optimal correction — or appropriate phase defined by initial time t_0. One can use the implicit MATLAB solver ode15i to do so. We compared the results with those from the iterative action minimizing method [6], a numerical scheme based on Newton’s Method. Figure 2 depicts agreement of the time-dependent optimal paths resulting from the two different numerical methods. One can now use the paths to analyze the extinction event in Figure 1 (on page 3). Additional detail for this example is available in our published work [3].

Much work remains pertaining to the analysis of rare noise-induced transition events in time-dependent models. For epidemic models, such as the aforementioned SIS model, knowledge of the optimal path enables discovery of the mean time to extinction from the endemic state, which is verifiable by stochastic realizations. Determining the mean time to extinction allows researchers to quantify the effect of various control schemes—including vaccination pulses and treatment—on infectious disease. More generally, one can use classical dynamical systems theory to understand noise-induced transitions in a wide variety of stochastic problems, from biology to physics. Some examples include genetic switching, pest eradication, and bit error rate in communication systems. As scientists increasingly discover more systems that exhibit rare events, we look forward to the implementation of old and new techniques to understand the effect of stochasticity and control measures.

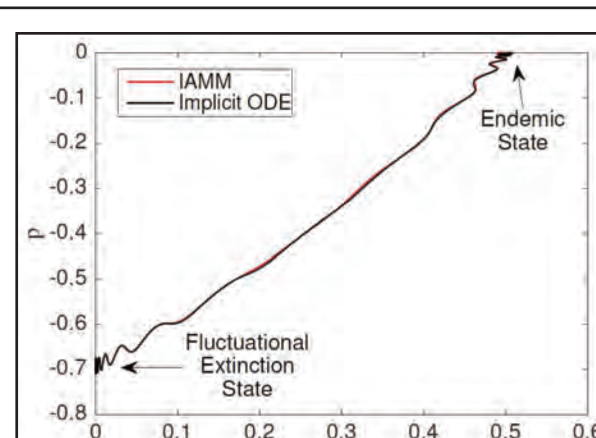


Figure 2. *The phase space for Hamilton’s equations with forcing, which corresponds to the stochastic SIS model with seasonality. Note the location of the endemic and fluctuational extinction states and the connecting manifold. Approximation of the optimal path found with the iterative action minimizing method numerical scheme appears in red. The implicit ordinary differential equation solution is plotted in black. Image courtesy of [3].*

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Lora Billings is dean of the College of Science and Mathematics at Montclair State University. Her research focuses on applied deterministic and stochastic dynamical systems that model applications in epidemiology, physics, and ecology. Eric Forgoston is an associate professor of applied mathematics at Montclair State University. His research involves the study of complex physical and biological phenomena, including material transport in the ocean, the outbreak and extinction of infectious diseases, behavior of biological and robotic swarms, food web dynamics in ecological systems, and the stability of fluid flows.

Multi-Agent Decision-Making in Design

By Paul Davis

Naomi Ehrich Leonard of Princeton University and her colleagues employ a bold approach when designing controls for multi-agent systems. "[We use] the tools of nonlinear dynamics to connect collective decision-making in animal groups with the design of collective decision-making in robotic groups," she says. For example, she asks if capturing such collective behavior could frame the development of a control system enabling turbines in a wind farm to respond collectively, similarly to a flock of birds. Could they react to changes in wind direction, given the constraints of their limited sensing, communication, and computation?

Instead of attempting to capture the system's essential *physical* properties, Leonard and her collaborators aim for a pitchfork bifurcation: the fundamental *mathematical* property of a multi-party decision-making dynamical system (see Figure 1a). The handle of a stable pitchfork bifurcation diagram represents deadlock; the twin forks that spring from it as the decision parameter changes signify the choices competing to attract consensus. The tools of singularity theory allow designers to "translate from nature to design and back again," Leonard says.

She adds that the systematic translation of the physical mechanisms that explain animal behavior to a "bio-inspired design methodology" is difficult. This is because most animal behavior studies are empirical rather than model-based, use simulation models that are not easily generalized, or rely on mean-field behavior that fails to reveal the effects of heterogeneity or system structure. Despite these challenges, Leonard and her team have identified the fundamentals of bio-inspired design for some remarkable examples of collective decision-making, such as that of house-hunting honeybees. After the birth of a new queen, the old queen and about half of her fellow bees abandon their hive in search of a new nest. Scout bees set off to investigate suitable nest sites. They report the direction and distance of sites that they find, promoting the sites' qualities with a sophisticated "waggle-dance" while competitors try to stop them with head butts (see Figure 2, on page 1). The house-hunting swarm reacts efficiently to the scouts when choosing the best site; the swarm can even reliably break ties between sites of near-equal value, in contrast to the befuddled house hunters on reality television.

To begin quantifying these ideas, Leonard explores a pair of differential equations for the fraction of the swarm preferring either site A or B. Figure 3 (on page 7) illustrates the transitions in this dynamic mixing model that lead to one possible steady-state outcome—a quorum committed to B with the remaining few either committed to A or undecided U—when site B is more valuable than A. In both the model and in nature, the swarm chooses the better site.

There are three possible outcomes in case of a tie in the value v of two sites: a lasting tie or a choice between one of the two locations (see Figure 4, on page 7). The difference between a tie and a decision depends on the strength of the scouts' commitment to their preferred sites. A tie's steady-state, which occurs when an equal number of scouts vote for A as for B, changes from stable to unstable when the bifurcation parameter σ—the strength of the stop signal, in this case the head butts—delivered to a scout for the other site increases from 0.2 to 5.

Figure 5 (on page 7) summarizes the honeybees' value-sensitive decision-making when a classic pitchfork bifurcation governs the stop signal's response. If the shared value of the two sites is relatively small, the scouts may wait for a third, better option or send a more intense stop signal σ and make a stronger commitment to render a tie unstable and force a stable decision for one site or the other.

This value-sensitive decision-making is also robust to perturbations in system parameters. Figure 1a shows the original pitchfork bifurcation for sites of equal value as the stop signal increases; 1b portrays the qualitatively similar diagram with saddle node bifurcation for sites of fixed, slightly different values; and 1c depicts the behavior as the difference in the values of

See **Decision-Making** *on page 7*

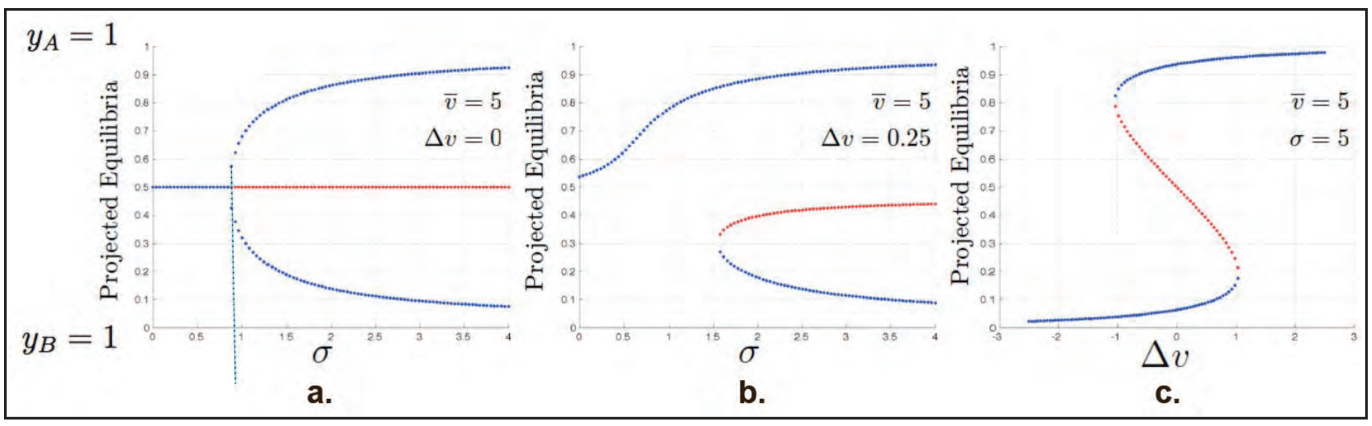

Figure 1. *Value-sensitive decision-making persists when a system is perturbed.* **1a.** *The original pitchfork bifurcation for sites of equal value as the stop signal increases.* **1b.** *The qualitatively similar diagram with a saddle node bifurcation for sites of fixed, slightly different values (unfolding).* **1c.** *Decision behavior with variation in the difference of the sites' values for a fixed stop signal. Image courtesy of [1].*

Forward-Looking Panel at MPE18 Examines the Intersection of Mathematics and Environmental Science

By Hans Engler and Emil Constantinescu

The second SIAM Conference on Mathematics of Planet Earth, sponsored by the Activity Group on Mathematics of Planet Earth (SIAG/MPE), took place last fall in Philadelphia, Pa. During the meeting, a forward-looking panel on emerging topics—in the style of a well-attended panel from the inaugural SIAG/MPE conference two years earlier—attracted a sizable crowd.

Panelists included Beth Wingate (University of Exeter), Mihai Anitescu (Argonne National Laboratory and The University of Chicago), Hans G. Kaper (Georgetown University), Christiane Rousseau (Université de Montréal), and Jon Hobbs (Jet Propulsion Laboratory). Emil Constantinescu (Argonne National Laboratory) moderated the discussion.

The panel's themes were derived from MPE-related work by mathematical scientists, and conversation covered the successful execution of such work and its potential focus areas in the future. Constantinescu opened the discussion by asking panelists about their particular success stories in accomplishing interdisciplinary work pertaining to MPE. He also conjectured how to best foster collaborations between mathematicians and domain experts.

All five panelists had interesting stories to tell and spoke about scientific efforts that involved substantial mathematics. Wingate explained how numerical analysts can aid and advance ocean flow modeling projects. Anitescu pointed to recent successes in high-quality forecasting (of weather, energy demand, etc.) that depend on mathematical contributions, while Kaper reported on a classical superconductivity modeling success where experiments confirmed mathematical predictions. Hobbs and Rousseau turned their attention to sustained interdisciplinary research involving mathematical scientists. Hobbs talked about the science team working with NASA's Orbiting Carbon Observatory 2 satellite—which collects spatiotemporal information related to environmental science—and Rousseau acknowledged high-level institutional collaborations through the International Science Council, which was founded in July 2018. The council's first president is a mathematician.

A question from the audience suggested that some MPE themes, such as "Planet Earth as a physical system" and "Planet Earth as a system at risk," might be more amenable to interdisciplinary collaboration than others, like "Planet Earth as a system supporting life" and "Planet Earth as a system organized by humans." Reluctant to agree with this assessment, panelists utilized the question to comment on expectations, rewards, and obstacles associated with interdisciplinary research. The typically long timeframes of such work make it challenging for junior researchers to get involved. Academic reward culture also tends to recognize contributions within the discipline, and our current training system may not adequately prepare students for interdisciplinary work on changing topics.

Constantinescu's second question to the group pondered a big MPE-related scientific challenge in need of resolution over the next five to 20 years, and how mathematics can play an influential role. Most panelists interpreted this question somewhat broadly and examined consequences stemming from global change. Hobbs specifically referred to a recent report entitled *Thriving on Our Changing Planet: A Decadal Strategy for Earth Observation from Space*[1] (published by the National Academies Press), which articulates a vision of scientific tasks for Earth sciences in the next decade. The report highlights the concept of Earth information and calls attention to major mathematical implications. Other panelists considered climate and environmental change and the pressing need to assess their inevitable effects, communicate relevant findings to the public, and develop and evaluate adaptation strategies. They also emphasized the obligation to understand extreme environmental events, possibly in nonstationary situations, and devise tools for interpreting small probability phenomena.

Another audience question broached ideas to improve effective interaction between academics and practitioners. Wingate remarked that the boundary between academic and nonacademic researchers is already blurring, at least in the United Kingdom. Rousseau pointed to past successful interdisciplinary and international research efforts that have had lasting scientific and political impact, including the International Geophysical Year (1957-1958) and the Intergovernmental Panel on Climate Change. A similar effort within mathematics was the MPE2013 initiative—supported by many international mathematical institutions and organizations—that led to the founding of SIAG/MPE.

The panel concluded after an hour of lively conversation. It provided a welcome opportunity for SIAG/MPE members with very different application areas to identify common scientific themes. We look forward to a similar panel at the next SIAM Conference on Mathematics of Planet Earth in 2020.

Panelists discuss present and future research at the 2018 SIAM Conference on Mathematics of Planet Earth, which took place in September 2018 in Philadelphia, Pa. From left: moderator Emil Constantinescu (Argonne National Laboratory), Mihai Anitescu (Argonne National Laboratory and The University of Chicago), Christiane Rousseau (Université de Montréal), Hans G. Kaper (Georgetown University), Beth Wingate (University of Exeter), and Jon Hobbs (Jet Propulsion Laboratory). Photo credit: Katie Kavanagh.

[1] https://www.nap.edu/catalog/24938/thriving-on-our-changing-planet-a-decadal-strategy-for-earth

Hans Engler is a professor of mathematics at Georgetown University. He previously chaired the SIAM Activity Group on Mathematics of Planet Earth. Emil Constantinescu is a computational mathematician at Argonne National Laboratory and scientist-at-large of the Consortium for Advanced Science and Engineering at The University of Chicago.

Inside Out: A Value-Based Approach to Industry Job Hunting

By Vrushali A. Bokil

Students looking for jobs in industry are often advised to develop soft skills, effective communication abilities, cross-disciplinary collaborative techniques, and interdisciplinary and transdisciplinary training. What jobseekers do not typically hear about is the need to understand *who they are*, *what they want*, and their *core values*, strengths, and passions. In other words, the most efficient method of career preparation is the *inside-out* approach. Roughly 50 young SIAM members received this message during the Professional Development Evening at the 2018 SIAM Annual Meeting, held in Portland, Ore., last July.

During the event, graduate students and postdoctoral researchers received advice and training from Di Ye, a professional trainer and certified coach at Zhennovate [3]. Ye provides high-performance training and one-on-one coaching to clients from various institutions, including Boeing, Microsoft, and the Massachusetts Institute of Technology. During a three-and-a-half-hour gathering, Ye—who has industry experience working at a variety of different firms—led the eager participants in a professional coaching session that comprised topics like "self-discovery and self-awareness," "clarifying your core values," and "strategic, authentic, and disciplined networking." The session also covered familiar subjects, such as writing a resume, developing and delivering an elevator speech, and preparing for a job interview. Ye explained the differences between industry and academia, the value of networking and referrals, and the realities of the job market. She discussed ways to determine goodness-of-fit for specific jobs and institutions, and articulated a big-picture view of the process of landing a satisfying "dream" job and fulfilling career, aided by a support network. For reference, attendees received a booklet entitled *Industry Job Hunting Bootcamp: How to Stand out in the Industry Job Hunting Process and Optimize your Career Potential*.

Di Ye of Zhennovate (left) interacts with attendees of the Professional Development Evening at the 2018 SIAM Annual Meeting, held in Portland, Ore., last July. SIAM photo.

To begin, Ye asked participants to clarify their core values—such as "authenticity" and "creativity"—and record a few of their most important principles along with a brief description of their personal significance. She emphasized that people must know who they are, what they want, and what their contributions are worth to confidently communicate and negotiate job offers with prospective employers. Effective communication is a crucial soft skill, and an elevator speech that employs powerful, concise statements and clear language helps one articulate his/her hiring potential. Participants had the opportunity to test what they had learned and practice their elevator pitches with volunteers from companies including Microsoft, NASA, 3M, and Nokia Bell Labs at a networking reception during the session.

The inside-out approach requires outside help in the form of a support network. Did you know that more than 70 percent of jobs on the market are not published [2]? Ouch! On average, every posted opening receives over 100 applications [2]. The realities of the job market require that applicants not only know who they are and what they want, but also have a network of people familiar with their values and passions to promote them to prospective job hunters. Ye classified members of this support network into four categories: people who provide emotional support, challenge one to grow, help expand one's possibilities, and share power. Participants then performed a short activity to identify the people in their network, pinpoint missing categories, and brainstorm ways to engage and expand their connections. Some attendees expressed discomfort in categorizing people this way, noting that it made friendships feel business-like. Ye assured them that this would not be the case, provided they approach networking with respect, authenticity, and a mindset that includes genuine intention and gratitude when building meaningful connections. This technique helps set the right expectation about the kinds of support to seek from each person in one's network.

Everyone wants to find their dream job — employment that is fulfilling, avoids undesirable tasks, and involves an ideal location. But is this always possible? One participant noted that applicants must face realities, such as paying bills. Ye reassured attendees that while their first job may not be their dream job, and they may have to work their way through several positions acting as stepping stones, they can eventually attain their ideal career. She also affirmed that realistically optimizing and strategizing one's job search is far more important than the quantity of applications he/she sends out. First salary does matter as a benchmark for future pay.

In summary, an inside-out approach that accounts for authenticity, curiosity, motivation to learn and contribute, willingness to build meaningful relationships, and constant adaptation to the job market will keep people moving towards their objectives and goals and hopefully lead to that ideal dream job.

The Professional Development Evening received positive feedback, and SIAM plans to offer similar career development

See **Industry Job Hunting** *on page 8*

Decision-Making

Continued from page 6

the sites varies for a fixed stop signal. Note the hysteresis in Figure 1c (on page 6) as the difference between the sites' values passes through zero. Instead of a sudden flip, the original choice persists until the difference in value becomes too large to ignore.

How might these ideas extend to increasingly realistic multi-agent systems with more heterogeneous agents and interconnections? For example, some agents may have pre-existing preferences or access to other external information. Or perhaps some bees are unable to see the waggle dance. Can the stop signal serve as a control input? Can such agents choose unanimously and robustly between competing choices?

Classic singularity theory for dynamical systems offers an answer through the pitchfork bifurcation's deadlock-or-decision property in Figure 1a (on page 6). An agent's decision is positive for choice A and negative for B; at consensus, all agents' decision variables are equal. The sum of the individuals' decision variables yields the collective decision.

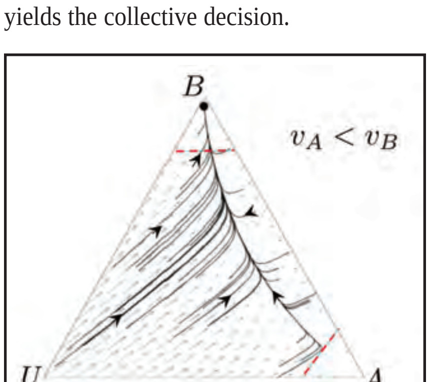

Figure 3. *Trajectories to the one steady-state outcome—a quorum committed to site B, and the remaining few either undecided U or committed to site A—for a swarm of honeybees deciding between nest sites A and B when B is of higher value than A and both are sufficiently valuable. Image courtesy of [1].*

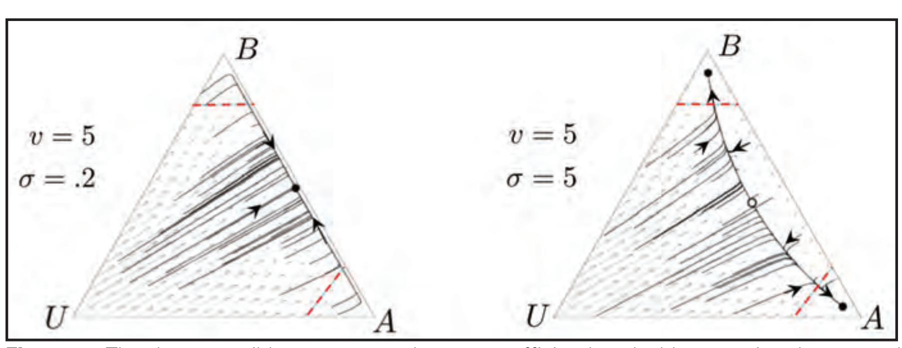

Figure 4. *The three possible outcomes when two sufficiently valuable nest sites have equal value v are either a tie or a choice between the sites. The difference between a tie and a decision is the strength σ of the scouts' commitment to their preferred sites. The tied steady-state—an equal number of the scouts voting for A as for B—changes from stable to unstable when the bifurcation parameter σ increases from 0.2 to 5. Image courtesy of [1].*

To formulate a model, suppose agent i's opinion x_i changes in response to external information (positive, negative, or zero); negative self-feedback; and positive, weighted, saturating "social" feedback from agents with whom it communicates. The social feedback is weighted by a control variable — a generalization of the stop signal. Critically, the (negative) sum of other agents' influence weights on a given agent is the latter's self-feedback factor; that is, an individual bee's opinion is influenced by a weighted sum of the differences between its opinion and a scaling—according to the control variable—of the saturating opinions of its neighbors in the network.

When the control variable is unity and the system is linearized about zero (no agent has an opinion), the system reduces to $\dot{x} = -Lx$, where L is the social network's rank-deficient Laplacian (or admittance) matrix. Network types might regard an agent's opinion as the potential x_i at its node. They might also conclude that the linearized system has a zero eigenvalue because a version of Kirchhoff's Voltage Law holds.

"We can create bifurcation by design," Leonard says. If the control variable is slightly bigger than unity, the first eigenvalue is positive and the steady-state undecided outcome—in which no agent has an opinion—is destabilized. As the control variable increases, a decision is taken. Furthermore, because "the center manifold is tangent to the consensus manifold, we get unanimity by design," Leonard explains.

Additional analysis reveals the effects of information and network asymmetry, explores value-sensitive decision-making, and builds a framework for feedback control of the bifurcation parameter (i.e., avoiding a deadlock among bees if they reach no consensus on a good potential nesting site).

How can humans design multi-agent control systems that match the performance of Mother Nature's networks among animals? Leonard's answer is to design a network using singularity theory to organize the agents' decision-making — bifurcation by design.

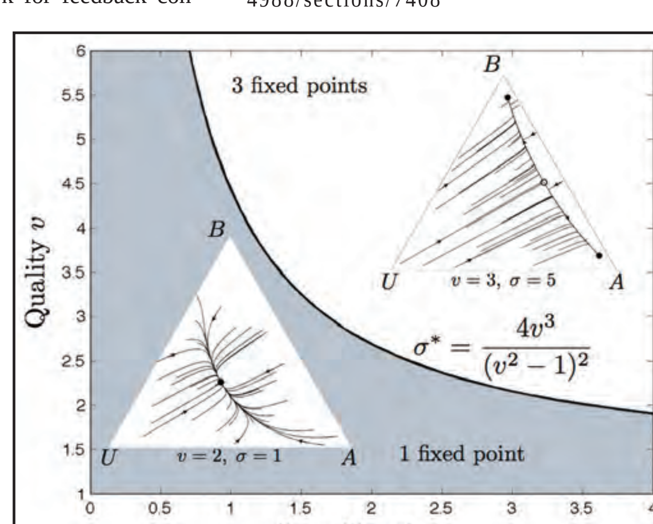

Figure 5. *Value-sensitive decision-making; a pitchfork bifurcation separates a tied vote between two sites of lesser but equal value (lower left) from a stable choice in favor of a particular site (upper right). Image courtesy of [1].*

Leonard delivered an invited lecture on this topic at the 2017 SIAM Annual Meeting. The presentation is available from SIAM either as slides with synchronized audio or a PDF of slides only.[1]

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[1] Pais, D., Hogan, P.M., Schlegel, T., Franks, N.R., Leonard, N.E., & Marshall, J.A.R. (2013). A Mechanism for Value-Sensitive Decision-Making. *PLoS ONE*, *8*(9), e73216.

Further Reading

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Paul Davis is professor emeritus of mathematical sciences at Worcester Polytechnic Institute.

[1] https://www.pathlms.com/siam/courses/4988/sections/7408

Brown University's TRIPODS Institute Furthers Research and Education in Data Science

By Jeffrey Brock, Alden Bumstead, and Björn Sandstede

Brown University's Data Science Initiative[1] is in its second year of running a TRIPODS institute. TRIPODS—Transdisciplinary Research in Principles of Data Science—is a program established and funded by the National Science Foundation (NSF) to "bring together the mathematics, statistics, and theoretical computer science communities to develop the theoretical foundations of data science through integrated research and training activities." Brown's institute, entitled "Foundations of Model Driven Discovery from Massive Data,"[2] is one of 12 TRIPODS institutes[3] across the United States.

The Data Science Initiative serves as a hub for research and education in the foundational methodologies, domain applications, and societal impacts of data science at Brown. A collaborative effort of Brown's Departments of Biostatistics, Computer Science, and Mathematics, as well as the Division of Applied Mathematics, the initiative was established in 2015 to foster methodological and domain-driven data science research and support educational and training activities in data science that intersect these disciplines. Thus, it was natural for our team—consisting of Jeffrey Brock (Department of Mathematics), Stuart Geman and Björn Sandstede (Division of Applied Mathematics), Joseph Hogan (Department of Biostatistics), and Eli Upfal (Department of Computer Science)—to apply for a TRIPODS grant.

Our institute's research is organized around three themes: geometric and topological methods for complex data, new tools for causal and model-based inference, and data analysis on massive graphs and networks. A concrete example of our ongoing work is an effort by Geman's team to improve collective understanding of deep neural networks: This research is based on the belief that one cannot substantially close the gap between human and machine performance with regard to interpretation (as opposed to classification) without architectures that support stronger representations. The group has developed a mechanism for embedding spatial and abstract relationships that bind parts and objects and define context within deep neural networks; currently the resulting dynamics considerably complicate both inference and estimation, although smaller experiments have been encouraging. A second project focuses on the development of algorithms that reveal differential genetic architecture between traits in sizeable genome data sets. The goal of this work is to use large genome-wide association studies that combine medical records with genome data from over 300,000 individuals to detect genes that share mutations across disease groups.

Training and community building have been major focal points of the institute over the past 18 months. Two activities were particularly successful: the first was an exciting weeklong summer bootcamp on "Topology and Machine Learning"[4] in August 2018, a feat that would not have been possible without TRIPODS. This program was hosted at the NSF-funded Institute for Computational and Experimental Research in Mathematics[5] and brought together 36 attendees from 20 institutions, ranging from undergraduate and graduate students to faculty and scientists from national laboratories. The bootcamp used tutorial lectures and hands-on labs to provide an in-depth introduction to topological data analysis, machine learning, and their intersection. At the end of the week, invited speakers presented their research in these areas and connected bootcamp topics to concrete research projects.

Lunch discussion during the TRIPODS Workshop on "Geometry and Topology of Data," held at the Institute for Computational and Experimental Research in Mathematics in December 2017. Photo credit: Alden Bumstead.

The second noteworthy event was a two-day informal networking workshop on "Building Community in the Foundations of Data Science,"[6] which took place that same month. The workshop involved a series of short talks and brainstorming sessions dedicated to strengthening data science activity in the Northeastern U.S. A mix of participants from a range of research universities and liberal arts colleges made the workshop informative, inspiring, and forward-looking. Discussions focused on research, education, and the need for accessible data-fluency curricula. Among the outcomes was a concrete list of suggestions for how TRIPODS could further research and educational activities in the Northeast, i.e., by developing listings of researchers, their expertise, and methods to encourage collaboration; organizing smaller working groups around specific topics; fostering the co-design of courses by instructors from different organizations; and supporting students and postdoctoral researchers capable of bridging multiple institutions.

We are excited about our three TRIPODS+X projects[7] that connect our TRIPODS institute at Brown with researchers from Duke University, Montana State University, Rutgers University, Smith College, Valparaiso University, and Yale University. One project aims to develop computational tools to identify gene regulatory networks from genome-wide expression data sets, another seeks to build methodological and theoretical bridges between neuroscience and data science, and a third attempts to improve data science education for undergraduates by recognizing student misconceptions about data science concepts and mismatches between core curricular elements and daily work of early-career practitioners.

More information about the 12 TRIPODS institutes and corresponding events is available at the TRIPODS website.[8]

Jeffrey Brock is a professor of mathematics and the inaugural Dean of Science in the Faculty of Arts and Sciences at Yale University. Alden Bumstead is associate director of the Data Science Initiative at Brown University. Björn Sandstede is a professor of applied mathematics, a Royce Family Professor of Teaching Excellence, and director of the Data Science Initiative at Brown.

[1] https://www.brown.edu/initiatives/data-science/
[2] https://www.brown.edu/initiatives/data-science/research/tripods
[3] https://nsf-tripods.org/institutes/
[4] https://icerm.brown.edu/tripods/tri18-2-tml/
[5] https://icerm.brown.edu/
[6] https://icerm.brown.edu/tripods/tri18-1-bcds/
[7] https://www.brown.edu/initiatives/data-science/news/2018/10/tripodsx-brown-university
[8] https://nsf-tripods.org/institutes/

Industry Job Hunting

Continued from page 7

workshops at future meetings. Stay tuned for announcements about these upcoming workshops from SIAM. In the meantime, online workshops about soft skills development are available through Zhennovate.[1] All job seekers in mathematics, statistics, and operations research would highly benefit from perusing *BIG Jobs Guide* by Rachel Levy, Richard Laugesen, and Fadil Santosa, now available for online purchase at the SIAM bookstore [1]. Jobs in these areas are projected to grow by 30 percent over the next decade. The guide provides practical steps for finding that dream job in business, industry, or government (BIG), and includes information on the types of available jobs as well as tips for writing high-impact resumes and securing rewarding internships. *BIG Jobs Guide* also offers insights to mentors and advisors on building institutional relationships and enabling math departments to aid students in finding BIG employment.

Acknowledgments: The author would like to thank Di Ye from Zhennovate, Fadil Santosa of the School of Mathematics at the University of Minnesota, and Richard Laugesen of the Department of Mathematics at the University of Illinois at Urbana-Champaign for feedback that greatly enhanced this article.

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[1] Levy, R., Laugesen, R., & Santosa, F. (2018). *BIG Jobs Guide: Business, Industry, and Government Careers for Mathematical Scientists, Statisticians, and Operations Researchers* (1st Ed.) Philadelphia, PA: Society for Industrial and Applied Mathematics.

[2] Smith, J. (2013, April 17). 7 Things You Probably Didn't Know About Your Job Search. *Forbes – Leadership*. Retrieved from https://www.forbes.com/sites/jacquelynsmith/2013/04/17/7-things-you-probably-didnt-know-about-your-job-search/.

[3] Ye, D. (2018, June). Stand Out in the Industry Job-Hunting Process and Optimize Your Career Potential. *SIAM News*, *51*(5), p. 10.

Vrushali A. Bokil is a professor in the Department of Mathematics at Oregon State University. She is a computational mathematician whose interests lie in mathematical modeling and numerical analysis, with applications in mathematical biology, epidemiology, and computational electromagnetics. Bokil is a member of the SIAM Career Opportunities Committee.

[1] www.zhennovate.com/events

Trump Administration Releases Update to STEM Strategic Plan

The National Science and Technology Council recently released the federal government's five-year strategic plan for science, technology, engineering, and mathematics (STEM) education. The report, *Charting a Course for Success: America's Strategy for STEM Education*,[1] represents a plan for STEM education programs, investments, and activities that are implemented by federal agencies. The 2018 plan is a five-year update of the initial 2013 plan released under the Obama administration. In addition to guiding federal activities and investments, the strategic plan notes its intention to serve as a "North Star" for the broader STEM community. The finalized plan includes several objectives that are relevant to this population:

Make Mathematics a Magnet: The plan notes that mathematics continues to serve as a gateway to higher-income jobs for STEM majors. Our comments were very supportive of mathematics' contextual integration across disciplines to improve mathematical literacy among students. This goal was ultimately incorporated, and the plan specifically emphasizes the importance of education in data science and modeling. Federal actions that the plan seeks to prioritize include support for programs and partnerships that integrate mathematics and statistics education in applied contexts, and education practices that demonstrate retention of diverse learners.

Make Computational Thinking an Integral Element of All Education: Consistent with our comments, the plan defines computational thinking (CT) broadly. The language specifically frames CT as "a set of broadly valuable thinking skills that helps people solve problems, design systems, and understand human behavior" and notes the strong linkage between CT and computer science. Policymakers and individuals in higher education are urged to integrate and standardize CT into learning materials and teacher preparation programs, while employers are encouraged to engage educational institutions to support CT instruction.

Encourage Transdisciplinary Learning: The strategic plan calls for further support of research, development, and dissemination of effective transdisciplinary STEM education practices, programs, and policies, as well as the expansion of support for STEM learners studying transdisciplinary problems through internships, fellowships, scholarships, and other training opportunities. It also promotes federal activities in support of the recruitment, preparation, retention, and upskilling of STEM educators in transdisciplinary approaches that integrate local and global community questions.

The strategic plan states the federal government's key role in working with stakeholders to further STEM education and supports three overarching goals: **Build Strong Foundations for STEM Literacy**; **Increase Diversity, Equity, and Inclusion in STEM**; and **Prepare the STEM Workforce for the Future**.

The plan identifies the following four pathways for achieving these goals:

1. Develop and Enrich Strategic Partnerships. This pathway is focused on strengthening and creating relationships between educational institutions, employers, and communities. For institutions of higher education, this involves contributing to STEM ecosystems that engage students in work-based learning, including internships, apprenticeships, and research experiences. The plan notes that "Having strategic partnerships also means exploring opportunities within the education community to blend formal and informal learning, and to blend curricula to enable students to complete both core academic and applied technical curricula in preparation for higher education."

2. Engage Students where Disciplines Converge. In this pathway, students are encouraged to engage with "real-world problems…using knowledge and methods from across disciplines." This path also issues a call to address the barrier that mathematics often creates in STEM career accessibility.

3. Build Computational Literacy. This pathway acknowledges the importance of digital literacy while calling for the advancement of CT, which "means solving complex problems with data." It also calls for expanded use of digital platforms for teaching and learning.

4. Operate with Transparency and Accountability. This pathway urges the federal government and stakeholders to utilize "open, evidence-based practices and decision-making" in monitoring progress towards the strategic plan's goals.

Each pathway has associated objectives for federal agencies and departments with STEM programs. These objectives are as follows:

- Foster STEM ecosystems that unite communities, increase work-based learning and training through educator-employer partnerships, and blend successful practices from across the learning landscape.
- Advance innovation and entrepreneurship education, make mathematics a magnet, and encourage transdisciplinary learning.
- Promote digital literacy and cyber safety, make computational thinking an integral element of all education, and expand digital platforms for teaching and learning.
- Leverage and scale evidence-based practices across STEM communities, report participation rates of underrepresented groups, use common metrics to measure progress, make program performance and outcomes publicly available, and develop a federal implementation plan and track progress.

SIAM leadership, particularly the SIAM Committee on Science Policy, will be planning its strategy to engage with agencies in 2019 in the context of this new plan.

The plan's vision is that "All Americans will have lifelong access to high-quality STEM education and the United States will be the global leader in STEM literacy, innovation, and employment." This vision—and the details set forth in the plan—are in line with themes that many policymakers have been sounding, including a global competition for scientific and technical talent as well as the relationship between STEM, economic prosperity, and national security. The plan recognizes education and research organizations as stakeholders that will engage federal agencies in supporting STEM education. It notes that "The United States has a higher education system that is the envy of the world, providing undergraduate and graduate degrees in STEM and conducting research that is an engine for American prosperity and security." Specifically-mentioned areas where further STEM education is needed to support the training of future researchers include national security, artificial intelligence, cybersecurity, quantum information science, and advanced manufacturing.

Next Steps

Federal agencies involved in STEM education will collaborate to develop a consolidated implementation method that includes additional necessary actions to meet the strategic plan's goals and objectives. Some agencies have already announced commitments related to the strategic plan's release. The National Science Foundation (NSF) stated that it will be working with other federal agencies in support of the *NSF Inclusion across the Nation of Communities of Learners of Underrepresented Discoverers in Engineering and Science* (INCLUDES)

See **STEM Strategic Plan** *on page 10*

[1] https://www.whitehouse.gov/wp-content/uploads/2018/12/STEM-Education-Strategic-Plan-2018.pdf

SIAM Announces Major CS&E Prizes

The following prizes will be awarded at the SIAM Conference on Computational Science and Engineering (CSE19), to be held February 25-March 1 in Spokane, Wash.

SIAM/ACM Prize in Computational Science and Engineering

Jack Dongarra of the University of Tennessee (UT) will receive the SIAM/ACM Prize in Computational Science and Engineering. SIAM awards this prize jointly with the Association for Computing Machinery (ACM) every two years for outstanding contributions to the development and use of mathematical and computational tools and methods for the solution of science and engineering problems. With this award, SIAM and the ACM recognize Dongarra for his key role in the development of software, software standards, software repositories, and performance and benchmarking software, as well as his community efforts to prepare for the challenges of exascale computing, especially in the adaptation of linear algebra infrastructure to emerging architectures.

Dongarra holds appointments as University Distinguished Professor of Computer Science in the Electrical Engineering and Computer Science Department, and director of both the Innovative Computing Laboratory and the Center for Information Technology Research at UT. He is also a Distinguished Research Staff member in the Computer Science and Mathematics Division at Oak Ridge National Laboratory, a Turing Fellow at Manchester University, and an adjunct professor in the Computer Science Department at Rice University.


Jack Dongarra of the University of Tennessee.

"The list of people who have been awarded this prize is truly amazing, and to be included on that list is humbling and a great honor, as it validates the work we have been doing in a major way," Dongarra said. "The work helps form the basic part of the fabric of computational science and high-performance computing. It can be directly integrated with many important technologies, which have uses in large-scale computations in medical and health sciences, high-performance computing for biomedical and biomechanical engineering, parallel computing in bioinformation and computational biology, modeling and simulation of materials sciences and processing controls, environmental sciences and physics, chemical and biochemical systems simulations, and other areas."

The research that won the prize was developed with support from the National Science Foundation and the U.S. Department of Energy. Dongarra's software is freely available to the community.

James H. Wilkinson Prize for Numerical Software

Jeff Bezanson, Stefan Karpinski, and Viral B. Shah of Julia Computing are the 2019 recipients of the James H. Wilkinson Prize for Numerical Software.

This prize recognizes innovative software in scientific computing developed by researchers in the earlier stages of their careers. To be eligible for the award, candidates must have worked in mathematics or science for no more than 12 years after receiving their Ph.D., allowing for breaks in continuity. The prize was established by Argonne National Laboratory, the National Physical Laboratory, and the Numerical Algorithms Group Ltd. in honor of James H. Wilkinson's outstanding contributions to the field of numerical software.

The award recognizes Bezanson, Karpinski, and Shah for the development of Julia,[1] an innovative environment for the creation of high-performance tools that enable the analysis and solution of computational science problems. Julia allows researchers to write high-level code in an intuitive syntax and produce code with the speed of production programming languages. The scientific computing community has widely adopted it for application areas that include astronomy, economics, deep learning, energy optimization, and medicine. In particular, the Federal Aviation Administration has chosen Julia as the language for the next-generation airborne collision avoidance system.

Bezanson conducted his Ph.D. thesis work on Julia, after which he co-founded Julia Computing with Karpinski to continue pushing the ideas and abstractions behind the software into new domains and bring the benefits of Julia to industry and academic users worldwide. Julia began as a hobby for Shah but has grown into a full-time preoccupation. He is currently the chief executive officer of Julia Computing.

The recipients expressed their delight upon winning the award. "We are very excited on a personal level to receive the 2019 James H. Wilkinson Prize for Numerical Software, but even more so for the Julia community as a whole," they said. "The prize is really a recognition of the community and accomplishments of the people developing amazing projects and libraries in Julia. The past winners of this prize have been a huge inspiration to us."

Bezanson, Karpinski, and Shah conveyed their gratitude to Alan Edelman, who—while not eligible for the prize—is the fourth co-founder of the Julia Project. "Without Alan's support and mentorship, Julia would have been extremely difficult to pull off," they said. "And it was already a long shot."

The trio also outlined Julia's offerings to society at large. "Julia brings down the time and cost for trying new ideas, inventing new algorithms, and creating new products," they continued. "Our community does research related to many of the grand challenges identified by the National Academy of Engineering, including climate change, affordable healthcare, clean energy, personalized medicine and education, and more. We stay attuned to our community and focus on building abstractions and capabilities in Julia that will help researchers achieve these greater goals. Enabling them and seeing the amazing things that they do with it gets us excited to work on Julia every morning!"

[1] https://julialang.org/


From left to right: Jeff Bezanson, Stefan Karpinski, and Viral B. Shah of Julia Computing.

STEM Strategic Plan

Continued from page 9

National Network, a program focused on diversifying the STEM workforce. Progress tracking is a major component of the strategic plan and will entail ongoing review of evidence from current programs, an annual inventory of STEM programs, and publication of participation rates by women, underrepresented minorities, and persons in rural areas in programs and activities. To support the use of evidence-based STEM processes, agencies are encouraged to identify and share effective STEM education programs, practices, and policies, including those at the postsecondary level and in lifelong learning. The plan urges institutions of higher education to use "the objectives as fruitful lines of scholarship, useful guidelines for course design, and touchpoints for teacher preparation programs."

The strategic plan notes that "There can be no doubt that STEM education continues to be a significant priority for the United States." It remains to be seen whether this sentiment will translate into increased funding and support for federal programs. Research organizations and institutions of higher education should use the plan's goals, pathways, and objectives to identify areas of strengths and expertise, and leverage those in recognizing and shaping future federal STEM opportunities. One should incorporate the language and themes employed in this plan when interfacing and engaging with policymakers. Efforts around rural communities, extension projects, increased participation by women and underrepresented minorities, digital platforms, and work-based learning will remain priorities for agencies engaged in STEM education. Lewis-Burke Associates, SIAM's Washington, D.C., liaison, will continue to monitor opportunities to shape the strategic plan's implementation.

Further Reading

[1] Committee on STEM Education of the National Science and Technology Council. (2018, December). *Charting a Course for Success: America's Strategy for STEM Education*. Washington, D.C.: National Science and Technology Council. Retrieved from https://www.whitehouse.gov/wp-content/uploads/2018/12/STEM-Education-Strategic-Plan-2018.pdf.

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— Lewis-Burke Associates LLC

Lie Brackets and Bulletproof Vests

MATHEMATICAL CURIOSITIES

By Mark Levi

A bullet punctures a wall made of two thin layers A and B of different materials (see Figure 1). Does the exit velocity change if we turn the two-layered "vest" inside out, as in the figure? It turns out that it does, and the difference $v_2 - v_1$ of exit velocities is given by a certain Lie bracket. Let us assume, perhaps unrealistically, that the bullet's deceleration is a given function of its velocity, $dv/dt = a(v)$; for example, one could take $a(v) = -kv^2$. The velocity considered as a function of the position x (rather than the time) satisfies the differential equation $dv/dx = f(v)$ with $f(v) = a(v)/v$. Indeed,

$$\frac{dv}{dx} = \frac{dv}{dt}\frac{dt}{dx} = \frac{a(v)}{v} = f(v). \qquad (1)$$

Similarly, for the second material we have $dv/dx = g(v)$ for some other function g. We thus have two vector fields, f and g, in the one-dimensional *velocity space*; the position x plays the role of time, since we eliminated t in (1).

I claim that the effect of swapping A and B is given by the Lie bracket:

$$v_2 - v_1 = (g'f - f'g)hk + \ldots = [g,f]hk + \ldots \qquad (2)$$

for small h and k. Here, $\ldots = o(h^2 + k^2)$.

The Lie bracket $[g,f]$ can be the difference between life and death (although this difference is small).

Proof

Let F^x denote the flow of the vector field f. In other words, v_0 becomes $F^x v_0$ after the bullet travels distance x in material A; the flow G^x is defined similarly for material B. Then

$$v_1 = G^k F^h v_0, \quad v_2 = F^h G^k v_0,$$

and

$$v_2 - v_1 = F^h G^k v_0 - G^k F^h v_0,$$

This difference is given by the right side of (2), as verified by a Taylor expansion of $G^x v_0$ and $F^x v_0$ to second order; for example,

$$F^x v_0 = v_0 + fx + \frac{1}{2} f' f x^2 + \ldots.$$

I omit further details.

An example

If $f(v) = -v^\alpha$, $g(v) = -v^\beta$, then we get $[f,g] = (\beta - \alpha) v^{\alpha+\beta-1}$, and so only for $\alpha = \beta$ is the "vest" reversible.

I conclude with a trivial but curious observation, worth mentioning in an ordinary differential equation or mechanics course or a faculty lounge: if $dv/dt = f(v)$, then $K = \frac{v^2}{2}$ satisfies $dK/dx = f(v)$. Thus, $dv/dt = dK/dx$. *Velocity changes with respect to time exactly like kinetic energy changes with respect to distance.*

As a special case of this remark, *the kinetic energy decays exponentially with the distance* for the commonly-assumed quadratic drag $f = -cv^2$. Indeed, we have $dK/dx = f(v) = -2cK$.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

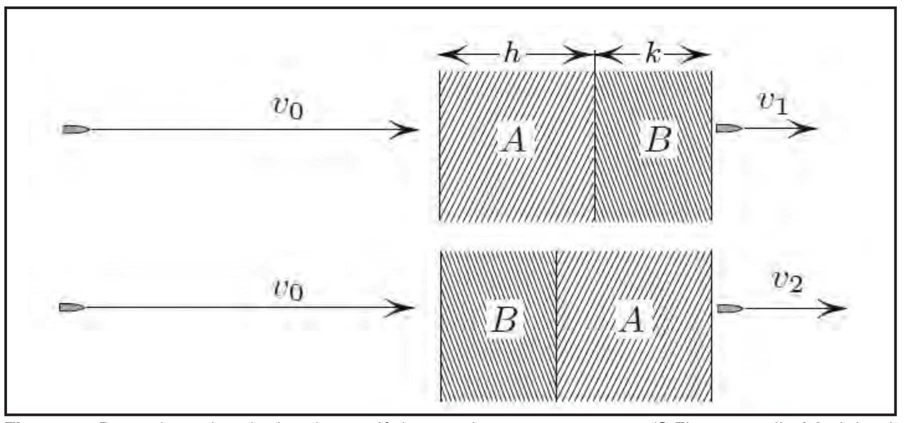

Figure 1. *Does the exit velocity change if the two layers are permuted? Figure credit: Mark Levi.*

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For rates, deadlines, and ad specifications, visit ***www.siam.org/advertising****.*

*Students (and others) in search of information about careers in the mathematical sciences can click on "Careers" at the SIAM website (****www.siam.org****) or proceed directly to* ***www.siam.org/careers****.*

Students Tackle Bayesian Inverse Problems in the Colorado Rockies

Reflections on the 2018 Gene Golub Summer School

By Omar Ghattas, Youssef Marzouk, Matt Parno, Noemi Petra, Georg Stadler, and Umberto Villa

The entire 2018 Gene Golub SIAM Summer School team poses in the shape of a Gaussian curve in Breckenridge, Colo. Photo credit: Georg Stadler.

Recent years have seen rapid growth in the volumes of observational and experimental data acquired from natural or engineered systems. How do we extract knowledge and insight about these systems from all of this data? This learning-from-data problem is at its core a mathematical inverse problem. That is, given (possibly noisy) data and a (possibly uncertain) "forward" model that maps parameters to data, we seek to infer parameters that characterize the model. Inverse problems abound in all areas of science, engineering, medicine, and beyond. Examples include inference of internal defects from scattered ultrasonic wave measurements, anatomical structures from X-ray computed tomography data, coalescence of binary systems from detected gravitational waves, ocean state from surface temperature observations, and subsurface contaminant plume spread from crosswell electromagnetic measurements.

Inverse problems are often ill-posed, i.e., their solutions may not exist or be unique or stable to perturbations in the data. Simply put, the data—even when large-scale—does not provide sufficient information to fully determine the model parameters. Non-uniqueness can stem from noise in the data or model, sparsity or redundancy in the data, or smoothing properties of the map from model parameters to observables. In such cases, uncertainty is a fundamental feature of the inverse problem; we wish to both infer the parameters and quantify the uncertainty associated with this inference. The ability to do the latter opens the door to powerful capabilities for model-based decision-making under uncertainty.

A group of students and instructors hike to the summit of Quandary Peak, one of the 14,000-foot mountains near Breckenridge, Colo., during the 2018 Gene Golub SIAM Summer School. Photo credit: Georg Stadler.

Bayesian inference provides a powerful framework for solution of inverse problems under uncertainty. However, when the forward model is expensive (as for partial differential equations (PDEs)) and the parameter dimension is large (as with discretized fields), Bayesian inversion becomes prohibitive with standard statistical methods. The last few years have seen the development of advanced mathematical and computational methods for Bayesian inverse problems governed by complex, high-dimensional forward models. To introduce graduate students to these recent advances, we organized the 2018 Gene Golub SIAM Summer School (G2S3) last June in Breckenridge, Colo., with the theme of *Inverse Problems: Systematic Integration of Data with Models under Uncertainty*.

The Bayesian inversion theme was very popular, as evidenced by the 257 applications we received from students around the world. The two-week summer school was sponsored by SIAM through a generous endowment from the estate of mathematician Gene Golub. The National Science Foundation's (NSF) Division of Mathematical Sciences provided additional support for U.S.-based students. Together these funds allowed 44 students to attend. However, limitations on available funding and meeting space meant that many excellent applicants were turned down. Participants formed a highly diverse group with respect to gender, geography, and research interests. They hailed from five continents with a wide range of backgrounds spanning applied math, statistics, engineering, and natural sciences.

The summer school lectures offered an integrated presentation of deterministic and Bayesian inverse theory and algorithms that first introduced ill-posedness and regularization; developed the ideas and tools for deterministic inversion via adjoint-based first- and second-order sensitivity analysis and numerical optimization; acquainted students with the Bayesian statistical framework for finite- and infinite-dimensional inverse problems; and finally explored linearized and sampling-based statistical solution methods, which built on several deterministic tools.

Concepts discussed in the morning lectures were put into practice and examined more thoroughly during hands-on laboratory sessions in the afternoon. These sessions utilized powerful open-source software that implemented state-of-the-art deterministic and Bayesian inversion methods (hIPPYlib, MUQ) and finite element solution of PDEs (FEniCS). The laboratory components featured cloud-based interactive tutorials via Jupyter notebooks, which mixed instruction and theory with editable and runnable code. The notebooks enabled students to run and modify sample codes through their web browsers, permitting rapid experimentation with different inverse problem formulations, discretizations, and solution algorithms. An NSF XSEDE cloud system called Jetstream deployed the software libraries via Docker containers, simplifying their installation and use. To broaden the summer school's impact and availability beyond just attendees, all lab material—including 16 tutorials and associated code—is freely available on the G2S3 website.[1]

The school culminated with team-based research projects in which participants employed the theory, algorithms, and software they had learned to tackle realistic and challenging inverse problems of their choice. The 11 teams presented their projects—ranging in areas such as photoacoustic tomography, incompressible flows, seismic imaging, elastography, heat conduction, and wave scattering—on the last day of the program.

The secluded mountain environment of Breckenridge provided a relaxed and inspiring atmosphere highly conducive to learning, collaboration, and idea exchange. Additionally, the Rocky Mountains created many opportunities for recreational activities like hiking, whitewater rafting, and mountain biking.

Seven of Gene Golub's eight most-cited papers treat methods that are important components of inverse problems (least squares optimization, saddle point solvers, singular values, and cross-validation). We think he would have approved of the theme of the 2018 G2S3! We thank Gene Golub's estate, SIAM, and the NSF for making this year's school possible. The students' enthusiasm and positive feedback has encouraged us to offer a similar event in the future.

Omar Ghattas is a professor in the Departments of Geological Sciences and Mechanical Engineering and the Institute for Computational Engineering and Sciences at the University of Texas at Austin. Youssef Marzouk is a professor in the Department of Aeronautics and Astronautics at the Massachusetts Institute of Technology. Matt Parno is a researcher at the U.S. Army Cold Regions Research and Engineering Laboratory. Noemi Petra is a professor of applied mathematics at the University of California, Merced. Georg Stadler is a professor at New York University's Courant Institute of Mathematical Sciences. Umberto Villa is a research professor in the Department of Electrical and Systems Engineering at Washington University in St. Louis.

[1] http://g2s3.com/labs/index.html

Congratulations to the 2017 and 2018 *SIGEST* Authors

Each issue of *SIAM Review* (*SIREV*) contains the *SIGEST* section, which spotlights an outstanding paper of general interest that has previously appeared in one of SIAM's specialized research journals. *SIGEST*'s purpose is to make the 13,000+ readers of *SIREV* aware of exceptional papers published in SIAM's various journals.

We would like to congratulate the following authors of the *SIGEST* selections that appeared in the 2017 and 2018 editions of *SIREV*:

Vol. 59, Issue 1
Full Waveform Inversion and the Truncated Newton Method
By Ludovic Métivier, Romain Brossier, Stéphane Operto, and Jean Virieux
Originally published in the *SIAM Journal on Scientific Computing*

Vol. 59, Issue 2
The Foucault Pendulum (with a Twist)
By Richard Moeckel
Originally published in the *SIAM Journal on Applied Dynamical Systems*

Vol. 59, Issue 3
Core-Periphery Structure in Networks (Revisited)
By Puck Rombach, Mason A. Porter, James H. Fowler, and Peter J. Mucha
Originally published in the *SIAM Journal on Applied Mathematics*

Vol. 59, Issue 4
Interactive Information Complexity
By Mark Braverman
Originally published in the *SIAM Journal on Computing*

Vol. 60, Issue 1
Second-Order Necessary Conditions for Stochastic Optimal Control Problems
By Haisen Zhang and Xu Zhang
Originally published in the *SIAM Journal on Control and Optimization*

Vol. 60, Issue 2
On the Metric s–t Path Traveling Salesman Problem
By Zhihan Gao
Originally published in the *SIAM Journal on Discrete Mathematics*

Vol. 60, Issue 3
Algorithmic Trading, Stochastic Control, and Mutually Exciting Processes
By Álvaro Cartea, Sebastian Jaimungal, and Jason Ricci
Originally published in the *SIAM Journal on Financial Mathematics*

Vol. 60, Issue 4
Semidual Regularized Optimal Transport
By Marco Cuturi and Gabriel Peyré
Originally published in the *SIAM Journal on Imaging Sciences*